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MAXIMUM LIKELIHOOD ESTIMATES OF POLAR MOTION PARAMETERS

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1. INTRODUCTION

The frequency Fc and quality factor Qc of the Chandler wobble, the earth's free nutation with a nearly 14 month period, are of considerable geophysical interest because they provide information about the earth's elastic and anelastic properties at a frequency well below the seismic band. Smith and Dahlen (1981) provide a thorough review of the relationships between Fc, Qc and the earth's physical properties.

Estimates of Fc and Qc are made from observations of polar motion, the movement of the rotation axis with respect to geographical coordinates. In this study we use the monthly polar motion series of the International Latitude Service (ILS) (Yumi and Yokoyama, 1980), for the period January 1900 through December 1978, with supplementary data for the period 1979-1985. The ILS data form the longest available series that has been reduced in a homogeneous way, and should provide the most reliable estimates. For the period 1979-1985, both the Satellite Laser Ranging (SLR) data (BIH annual reports) and optical astrometry data (International Latitude Observatory of Mizusawa [ILOM]) are available. We smoothed and then interpolated both SLR and ILOM series with a cubic spline to obtain pole positions at the same intervals as the ILS series. The results fromthe combined ILS/SLR and ILS/ILOM series were essentially identical, and the remainder of this paper. will refer to the results obtained with the ILS/ILOM series.

We use estimators that were developed by Jeffreys in two papers appearing in 1940 and 1968. The symbols Fc and Qc denote the true values of the polar motion parameters, while F and Q indicate estimates. We refer to the estimator from the 1940 paper by the Roman numeral I, and to estimates derived from it as F(I), Q(I). Similarly, F(II) and Q(II) refer to the estimators in Jeffreys 1968 paper. Both I and II were developed from maximum likelihood arguments, assuming that a Gaussian random process is the cause of polar motion near the Chandler frequency. While this may not be correct, Monte Carlo experiments demonstrate that it is probably not a critical assumption, particularly for estimator II. The Monte Carlo experiments also permit an evaluation of estimator bias and variance in the presence of noise.

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2. AUTOREGRESSIVE DESCRIPTION OF POLAR MOTION

The discrete polar motion equation (Wilson, 1985) is the foundation for understanding Jeffreys' maximum likelihood estimators. This equation relates time samples of pole position, M, to time samples of the excitation axis position, X. Using the complex pole coordinate description in which the real part is associated with motion along the Greenwich meridian, and the imaginary part with motion along 90 degrees east longitude, the equation is

$$M_t = R\, X_{t-1/2} + S\, M_{t-1} \tag{1}$$

where

$$R = \frac{WT}{i\ \exp(-i\pi FcT)}$$

$$S = \exp(iWT)$$

$$W = 2\pi Fc(1+i/2Qc)$$

Fc is the Chandler frequency expressed in cycles per year (cpy), and Qc is the dimensionless quality factor, proportional to the exponential decay time Qc/π Fc. T is the time interval between observations, measured in years, and t is the time index which takes on integer values. An equation similar to (1) appeared in Jeffreys' 1940 paper, differing only in the value for the constant R, and the time index of X, the excitation series. The choice of R in (1) produces X in the same units and coordinate system as M. Equation (1) is a discrete version of the governing differential equation based on Euler's rigid body equations, and provides a very good approximation for the case of the ILS data where T= 1/12 year.

Estimating Fc and Qc requires that we find the central frequency and width of a spectral peak from a finite length time series, a problem that has received a great deal of attention in geophysical, time series, and electrical engineering literature. Although there exist many methods to solve this type of problem, some are clearly inappropriate. For example, if X is a broad band process, then polar motion has a continuous rather than discrete spectrum near the Chandler frequency, and will not be strictly sinusoidal in time. Thus, fitting sinusoids (harmonic analysis), which is a suitable method for line spectra, as found in tidal studies, for example, is inappropriate for the Chandler wobble.

Autoregressive spectral analysis, often called Maximum Entropy Spectral Analysis (MESA) (Ulrych and Bishop, 1975) seems particularly appropriate because equation (1) shows that the polar motion time series is an autoregressive process of order 1 (AR-1) if X is a Gaussian random process and the data are free of noise. Unfortunately, a noisy polar motion time series is no longer a simple AR-1 process (Box and Jenkins, 1970), and observed polar motion contains a drift and annual component which do not conform to the AR-1 model. Following Jeffreys we retain the AR-1 model by removing the drift and the annual term, and applying a correction for noise. This preserves the simplicity which permits analytical expressions for both the estimates and their uncertainty. Alternatively, one could allow the order of the AR process to increase to accommodate the additional variance introduced by noise, drift, etc, which is equivalent to lengthening the Prediction Error Filter in the MESA (Vicente and Currie, 1976); or one might introduce a more complicated Auto Regressive-Moving Average (ARMA) model to include the effects of noise and drift (Ooe, 1978 and Wilson, 1979).

Assuming the AR-1 model to hold, any linear combination of M is also an AR-1 process. The simplest linear combination is an average of N adjacent samples

$$\overline{M}_t = \frac{1}{\sqrt{N}} \sum_{k=t}^{t+N-1} M_k \tag{2a}$$

The square root (N) normalization keeps the standard deviation of the noise in the average equal to that in the original data. Time samples of non overlapping averages will obey equation (2b), which is similar in form to (1):

$$\overline{M}_t = R\langle X \rangle + S^N M_{t-N} \tag{2b}$$

where $\langle X \rangle$ denotes an appropriate average of the excitation over the interval. Equation (2) is the basis for estimator I.

Another linear combination of M is the Fourier coefficient at the frequency of +6/7 cpy (=0.8571 cpy) as determined from a 14 month segment of the monthly time series. The motivation for using this type of average is that the Fourier series coefficient is a narrow-band filtered version of the data, and thus will capture the signal of the Chandler wobble, while rejecting most of the broad band noise. For the ℓth segment this coefficient is

$$A_\ell = \frac{1}{\sqrt{N}} \sum_{t=14\ell}^{14\ell+13} M_t \exp(-2\pi i t/14) \tag{3a}$$

where the square root normalization again provides the same standard error in A_ℓ as in the original data. The sequence of these Fourier coefficients determined from adjacent non-overlapping 14 month segments obeys an equation similar to (2), and adjacent coefficients in segments , -1 are related by

$$A_\ell = R\langle X \rangle + S^\ell A_{\ell-1} \tag{3b}$$

which is of the same form as (2) or (1), except that now $\langle X \rangle$ is an appropriate average of the 6/7 cpy Fourier series coefficients of the excitation function. Equation (3) forms the basis for estimator II.

3. MAXIMUM LIKELIHOOD ESTIMATES

The annual motion and the slow drift must be removed because they do not conform to the AR-1 model. We use Jeffreys' (1968) procedure to remove the drift: the data are divided into subsets containing seven years of data, which is exactly 7 annual periods and close to 6 Chandler periods;the mean value over each 7 year period is computed; a cubic spline interpolater is applied to each component of the 7 year means to obtain monthly values of the drift; finally, the interpolated drift series is subtracted from the original data. Figure 1 shows the drift computed in this way. After subtracting the drift, the annual component is determined by finding the best least squares fit sinusoid at a frequency of 1 cpy. In units of milli arc seconds (mas) the annual term is

$$(-45.4 \cos(+) - 84.2 \sin(+)) + i(72.5 \cos(+) - 31.7 \sin(+))$$

or in terms of prograde and retrograde components

$$(-38.6 + i78.3)\exp(i+) + (-6.8 - i5.9)\exp(i-)$$

The arguments of the trigonometric functions are radians after after January 1, with the symbols (+) or (-) indicating the sign of the argument. After subtracting the drift and annual terms, the remaining series, shown in Figure 2, is assumed to be the Chandler wobble arising from a Gaussian random excitation, with added noise which is independent from month to month.

In the maximum likelihood method (MLM), it is assumed that $\langle X \rangle$ forms a sequence of zero-mean Gaussian random numbers with independent real and imaginary parts. This is reasonable because even if individual values of X are not Gaussian, the averages, $\langle X \rangle$, will tend to be by the Central Limit Theorem. Furthermore, independence among the sequence of non-overlapping averages should improve with increasing N, even if adjacent values of X are not independent. The MLM estimates correspond to those values of F and Q which minimize the variance of the series $\langle X \rangle$ determined from the data using equations (2) or (3), because least squares and maximum likelihood are equivalent for Gaussian random variables. Since the structure of equations (2) and (3) is identical, we present the explicit expressions using $\bar{M}$ to refer either to the simple average in (2) or to the Fourier coefficients in (3).

The estimates depend on the data through the variance U, and the covariances V and W.

$$U = \sum_{t=1}^{p} |M_t|^2 - 2ps^2 \tag{4}$$

$$V + iW = \sum_{t=2}^{p} M_t M^*_{t-1} \tag{5}$$

The number of independent values of $\bar{M}$ is denoted by p, and U has been corrected by subtracting the contribution of the noise, which has standard deviation s in each component of the complex datum. No noise correction is applied to V or W, because they incorporate products from non-overlapping intervals with presumably uncorrelated errors. Following Jeffreys, we define

$$a = V/U \qquad b = W/U$$

and the MLM estimates are

$$F = 1/(2\pi TN)\ \mathrm{Tan}^{-1}(b/a) \qquad 1/Q = \ln(a^2 + b^2)/(-2\pi FTN) \tag{6}$$

For estimator II, (equation 3), F is the correction to the trial frequency of 6/7 cpy. Standard errors for a and b are given by

$$s_a^2 = s_b^2 = (1 - a^2 - b^2)/2p \tag{7}$$

and may be used to calculate the corresponding errors of the estimates. Because 1/Q tends to be normally distributed, standard errors are first calculated in terms of 1/Q, and then the corresponding limits for Qc are determined from their reciprocals.

Table 1 shows estimates and standard errors obtained with estimator II, and estimator I with N=1,2,3. F(I), F(II), and associated standard errors are practically independent of the value assumed for s, but Q(I) and Q(II) depend strongly on s. The best modern optical determinations have standard error s of about 10 mas. For the ILS data, Jeffreys (1968) estimated s to be near 30 mas while Wilson (1979) found s to be about 28 mas. Estimates of Qc are shown for s = 10 and 30 mas. Table 1 shows that estimates obtained by methods I and II are generally inconsistent in that their confidence intervals do not overlap in most cases. Thus, one or both of the estimators must be at fault, and we turn to Monte Carlo experiments in the next section to discover the source of the inconsistency.

Table 1 - Polar Motion Parameter Estimates
From the ILS/ILOM Series 1900-1985

Method and N	F (cpy)	Q(s=10mas)	Q(s=30 0 mas)
I-1	0.8178 +/- .0116	10 (8,13)	57 (37,114)
I-2	0.8263 +/- .0067	15 (12,21)	35 (25,57)
I-3	0.8322 +/- .0106	18 (14,25)	26 (25,57)
II-14	0.8436 +/- .0022	123 (70,471)	134 (75,592)

4. MONTE CARLO EVALUATION OF ESTIMATORS

Estimates obtained by methods I and II are complicated functions of the data, and involve only approximate corrections for the effects of noise. Thus it is not easy to analytically predict their performance, as measured by bias and variance, especially in the presence of noise. For this task, we turn to Monte Carlo studies using simulated polar motion data created with a random number generator. With simulated data, the true polar motion parameters are known exactly, and we may compare the estimates from I and II with the known values. The Monte Carlo experiments can also test the performance of the estimators when the data deviate from the AR-1 model.

The simulated polar motion data were generated from Gaussian, zero mean, random numbers at 1 month intervals. The standard deviation of the excitation was chosen to be 15 mas in each component of X, based upon estimates of the excitation power spectrum near the Chandler frequency by Wilson and Haubrich (1976). The monthly values of X were used to generate the simulated polar motion series using equation (1), with an initial pole position amplitude that was a uniformly distributed random number between 50 and 150 mas. Fc in equation (1) was fixed at 0.843 cpy, and Qc was set at 50, 100, or 200. The Gaussian noise added to the simulated polar motion series was given standard deviations varying between 0 and 40 mas in each coordinate.

The Monte Carlo Experiments were performed on ensembles of 50 independent series of 1032 points, corresponding to 86 years of monthly data. The 50 estimates of Qc and Fc were used to determine estimator bias and standard deviation. The average predicted standard deviations from equation (7) were also computed. The most significant finding from these experiments was that Method II was far superior to I regardless of the noise level and the value of Qc. For example, standard deviations of F(II) were an order of magnitude or more smaller than those for F(I). For non-zero noise levels, Q(I) was very biased, yielding values that were an order of magnitude too small. This is consistent with the very low values for Q(I) shown in Table I. Q(II) was also biased by noise, as described below.

Additional Monte Carlo experiments were conducted in which X consisted of a white noise series which had been integrated once or twice in time. This represents a departure from the AR-1 model, in that adjacent values of X are no longer independent. The performance of estimator II was found to be unchanged, and thus we conclude that the assumption that monthly values of X are Gaussian and independent is not critical.

While F(II) is unbiased by noise, Q(II) becomes biased as the noise level increases, as shown in figure 3. The bias is approximately independent of the value of Qc. On the basis of this figure, we propose a correction for the estimate in Table I by the factor 1.33, and a corresponding increase in the upper limit on the confidence interval for Qc. To be conservative, we retain the original lower confidence limit on Qc. The Monte Carlo experiments also show that when s=30 mas, equation (7) predicts standard errors for F(II) and Q(II) which agree reasonably well with the observed standard deviations in the Monte Carlo experiments.

5. DISCUSSION AND CONCLUSIONS

Table 2 summarizes the estimates obtained in this study by method II, including the adjustment for the bias in Q as discussed above. Table 2 also summarizes a number of other published estimates.

Estimates obtained by method I are quite similar to those reported by Jeffreys (1940), and in both cases are inconsistent with method II results. This inconsistency is attributable to the poor performance of method I, demonstrated by the Monte Carlo experiments. The superior performance of method II is due to the fact that the Fourier coefficients derived from 14 month segments of the data at 6/7 cpy have vastly better signal to noise levels than an individual datum. The Fourier coefficient is effectively a narrow-band filtered version of the data which preserves that portion of the data with the best signal to noise level. Method II results obtained in this study are consistent with Jeffreys (1968) results and with other recent estimates.

Method II is probably suitable for more general spectral analysis problems in which the frequency and width of an isolated peak are to be estimated in the presence of noise. Method II constitutes a compromise between traditional Fourier analysis and autoregressive spectral methods, using the Fourier series as a narrow band filter to first reject the noise and then autoregressive analysis as a sensitive high resolution estimator of the central frequency and spectral line width.

Table 2
Polar Motion Parameter Estimates

Source	Method	F	Q
Jeffreys (1940)	I	0.8177 +/-.0127	46 (37,60)
Jeffreys (1968)	II	0.8432 +/-.0043	61 (37,193)
Ooe (1978)	ARMA	0.8400 +/-.0039	96 (50,300)
Wilson&Vicente (1981)	ARMA	0.8430 +/-.0070	175 (48,1000)
This study (bias corrected)	II	0.8435 +/-.0022	179 (75,789)

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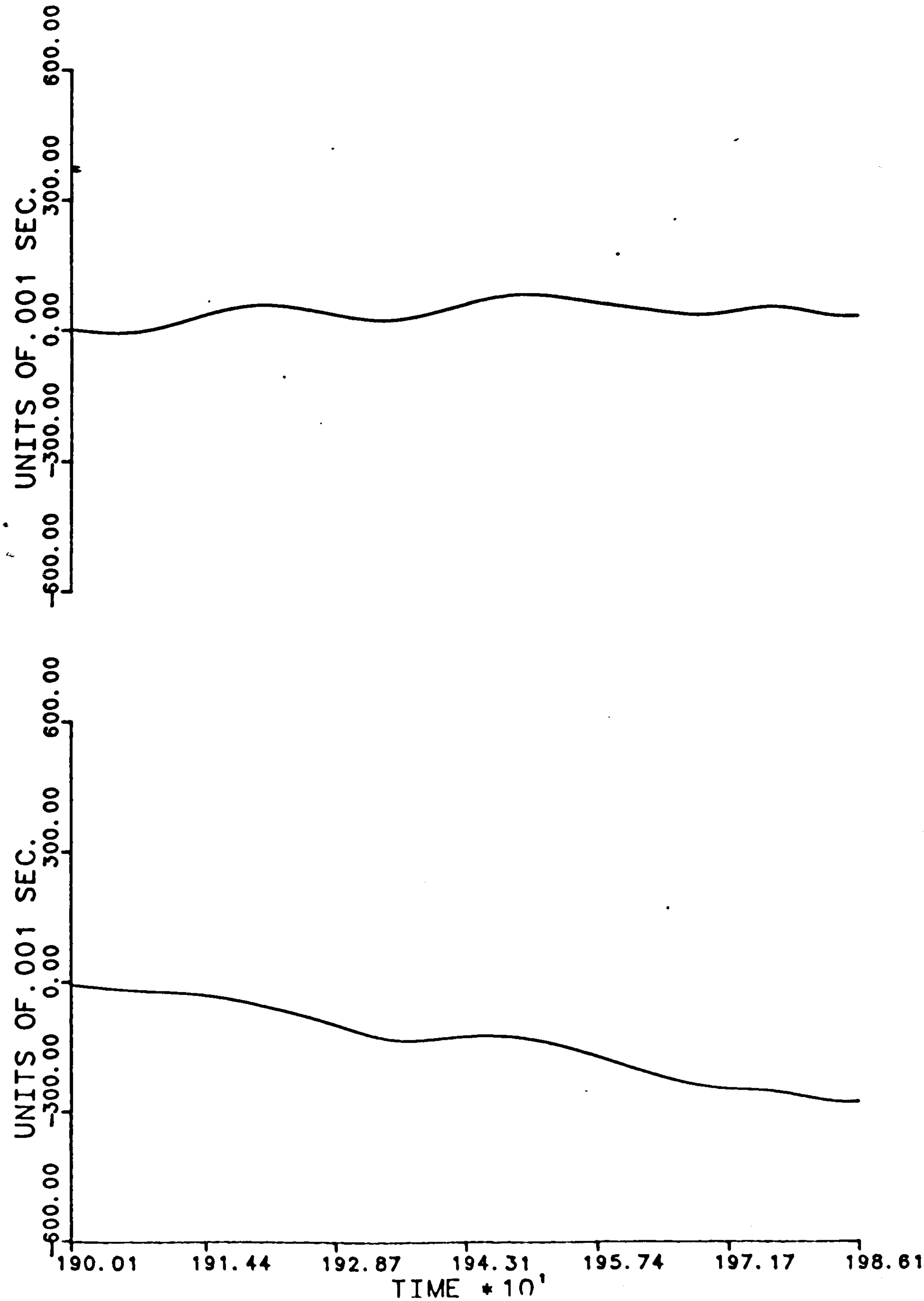
FIGURE CAPTIONS

Figure 1: The drift removed from the ILS/ILOM series as determined from means over 7 year intervals and cubic spline interpolated to monthly values.

Figure 2: The ILS/ILOM series after removal of drift and annual components.

Figure 3: The ratio of estimated Q by method II to the true value as a function of noise level. Each symbol represents the result of an experiment in which an ensemble of 50 simulated polar motion series were used to obtain average values of 1/Q(II), as described in the text.

Fyure 1



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192.87
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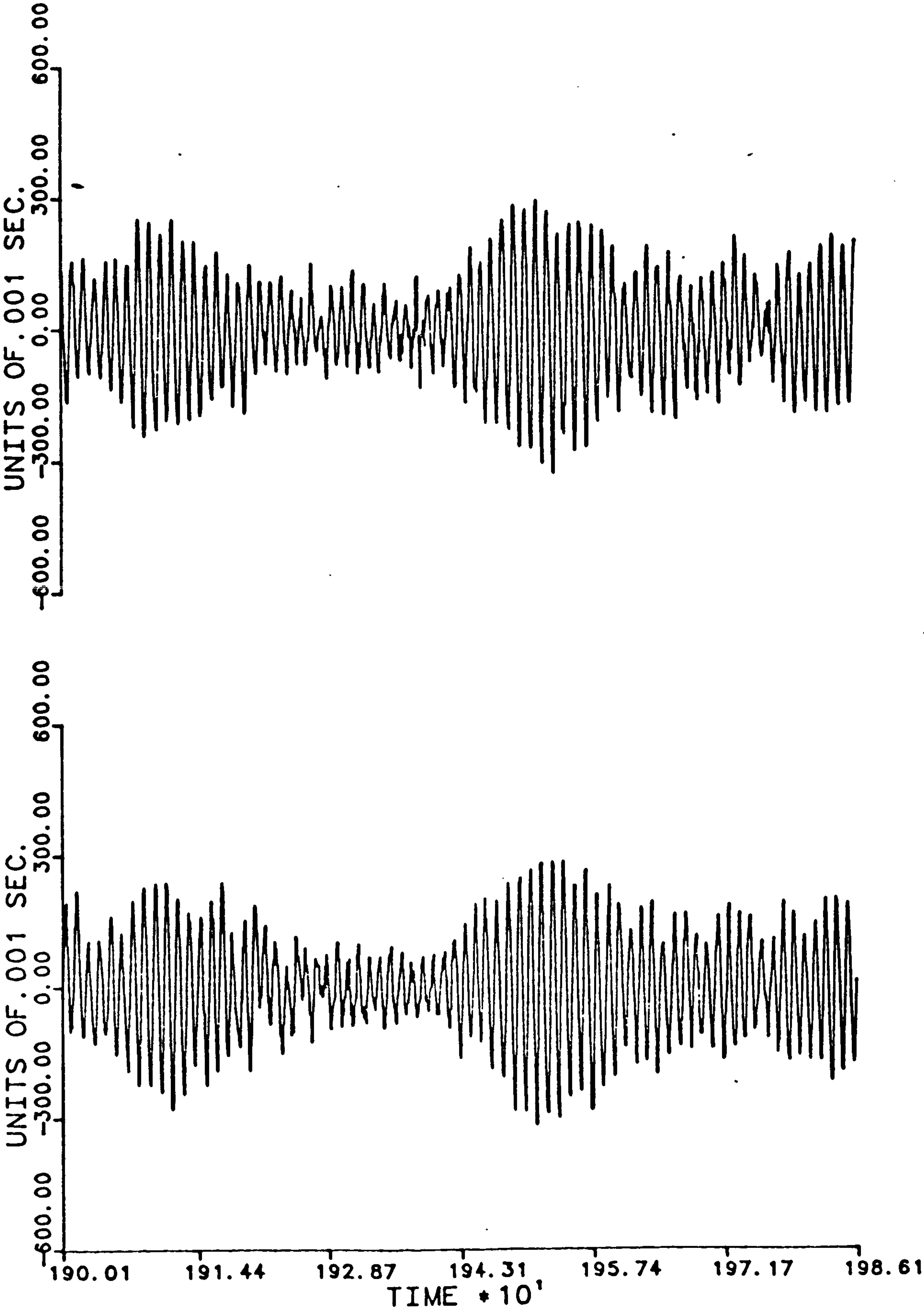
197.17
198.61
TIME * 10¹

Figure 2



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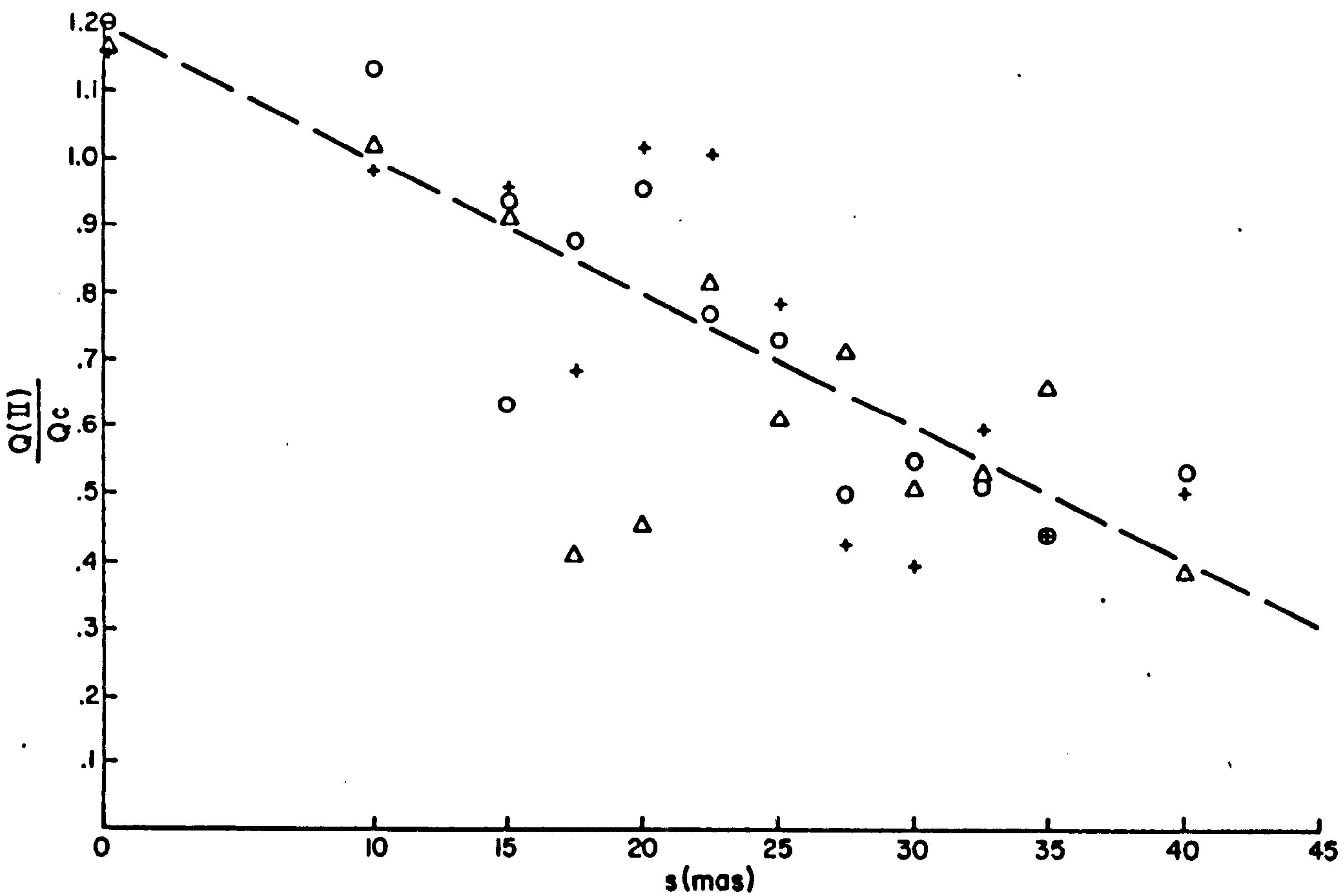

$\frac{Q(II)}{Q_c}$
1.20
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
10
15
20
25
30
35
40
45
s (mas)